Estimating the Elastic Constants of a Pultruded Profile

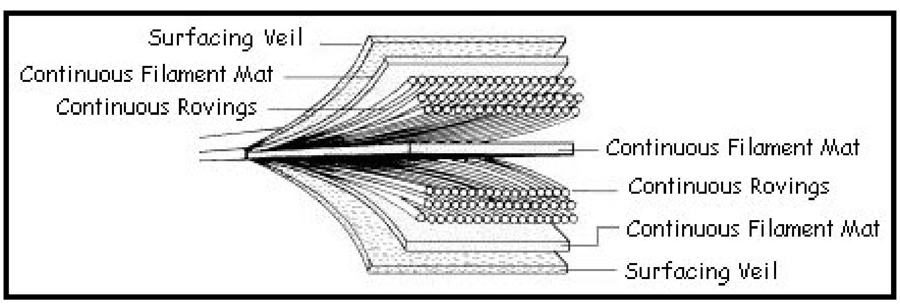
Pultrusion is the only process for continuous manufacture of a thermoset-based composite. The process involves pulling a package of fibre reinforcement from a storage creel through a bath of catalysed resin, then through a series of squeeze-out bushes and, finally, through a heated steel die wherein the resin crosslinks. A prerequisite of the process requires a profile to have a constant cross section. Typical profiles include I-beams, channels, solid rods and tubes.

The ingredients of a pultrusion precursor may include resin, catalyst, release agent, pigment, particulate filler and reinforcing fibre. In this description only the (catalysed) resin and the fibre reinforcement are considered.

Almost all thermoset resins may be used in pultrusion. The most common include unsaturated polyester, epoxy, phenolic, vinyl ester, and urethane acrylate. Reinforcements include E-, S- and R-glass, carbon, aramid and natural fibres such as jute and flax. Combinations of E-glass fibre and unsaturated polyester account for the great majority of profiles used in the building and construction sector while carbon/epoxy systems are widely used for aerospace and military applications. This discussion focuses on the former class of materials.

A typical “lay-up” for a pultrusion profile is shown in Figure 1. It consists of a number of layers or plies of different style of glass reinforcement. In this case these include continuous filament mat (CFM), continuous roving (CR) and a surfacing veil. The contribution of the latter to the mechanical properties of the profile may be ignored.

Figure 1: CR/CFM Pultruded Laminate Construction



On the basis of Figure 1, it is reasonable to consider the profile as a laminate that is made up of a number of layers (laminae) each of which – in this case - may incorporate a different style of reinforcement. In order to calculate the elastic constants of the whole profile it is first necessary to calculate the elastic constants of each lamina and manipulate these according to the volume fraction of each lamina in the profile. Note that the elastic constants of the profile do not depend on the stacking sequence of the laminae.

The elastic constants of a lamina are determined by the particular style of reinforcement since this defines the volume fraction of fibres in the lamina and their orientation.

The first step in this sequence is to determine the maximum volume fraction of fibre in each lamina, ϕ_{max} . This depends on the compressibility of the lamina and the pressure in the pultrusion die. Experimentally, this relationship may be expressed as

$$\phi_{max} = \alpha + 40\beta \tag{1}$$

The constants, α and β are given in Table 2 for a number of styles of reinforcement. These data are based on a pressure of 1.6kg/cm^2 (0.157 MPa = 1.57 bar = 22.75 psi) [1]

Table 1: Compressibility Constants for Reinforcing Fabrics

FABRIC	α	β
GLASS		
Continuous Filament Mat (CFM)	8.2	0.49
Chopped Strand Mat (CSM)	20	0.46
Continuous Roving (CR)	32	0.75
Woven Roving (WR)	20.9	0.61
CARBON		
Continuous Tow (CTC)	33.8	0.8
+/-45^o Fabric (CF)	35	0.51
Tissue (CT)	3.4	0.53
OTHER		
Aramid Fabric (AF)	46.9	0.51
Glass/Carbon Tape (HGCT)	31.9	0.74

The mass fraction of fibre, μ_f is related to the volume fraction, ϕ_f, by

$$\frac{1}{\mu_f} = 1 + \frac{\rho_r}{\rho_f}\left(\frac{1}{\phi_f} - 1\right) \tag{2}$$

For the more common glass reinforcements, the maximum volume fractions and mass fractions of reinforcement are given in Table 2.

Table 2: Maximum Volume and Mass Fractions in Various Laminae

Material	CR	WR	CFM
ϕ_{max}	0.62	0.45	0.28
μ_{max}	0.79	0.66	0.49

Using the various Rules of Mixtures given in Table 3, together with the material properties given in Tables 4 and 5, the elastic constants for UD laminae at the maximum volume fractions of Table 2 may be calculated. These are shown in Table 6.

Table 3: Rules of Mixtures for the Elastic Constants of a UD Lamina

Parallel, Iso-Strain Model	$E_{11} = \phi_f E_f + (1-\phi_f) E_m$
Series, Iso-Stress Model	$\dfrac{1}{E_{22}} = \dfrac{\phi_f}{E_f} + \dfrac{(1-\phi_f)}{E_m}$
Shear	$\dfrac{1}{G_{12}} = \dfrac{\phi_f}{G_f} + \dfrac{(1-\phi_f)}{G_m}$
Major Poisson's Ratio	$\nu_{12} = \phi_f \nu_f + (1-\phi_f) \nu_m$
Minor Poisson's Ratio	$\nu_{21} = \nu_{12} \dfrac{E_{22}}{E_{11}}$

For completeness, the shear modulus of resin and fibre may be found from

$$G_{f,r} = \frac{E_{f,r}}{2(1+\nu_{f,r})} \qquad (2)$$

Table 4: Properties of Thermoset Resins

Property	Unsaturated Polyester	Vinyl Ester	Epoxy
Tensile Strength, MPa	77	81	76
Failure Elongation, %	4.5	5	6.3
Poisson's Ratio	0.33		
Flexural Strength, MPa	123	138	115
Flexural Modulus, GPa	2.96	3.72	3.24
TDUL, °C	71	104	165
Relative Cost, £/kg	1	3	5-7

Table 5: Properties of Reinforcing Fibres

Fibre	Density, kg/m^3	Young's Modulus, Gpa	Tensile Strength, GPa	Failure Strain, %	ν_{12}
E-glass	2450	72	2.0-3.5	2.5	0.22
S-glass	2490	86	4.57	2.8	
R-glass	2750	85	4.14		
Aramid	1440	63-67	3.0-3.15	3.0-3.7	
Aramid-HM	1450	120-130	2.7-4.0	2.0-3.0	
Carbon-XA	1800	230-240	3.4-4.5	1.5-2.0	
Carbon-IM	1750	290-300	3.4	1.1-1.5	
Carbon-IM	1770	305	5.5	1.8	
Carbon-HM	1860	360-405	2.7-3.1	0.7-0.8	
Carbon-HS	1800	270	5.5	2.0	
Carbon-HM	2180	827	2.2	0.27	

Table 6: Elastic Constants for UD Laminae based on E-Glass and Unsaturated Polyester

Lamina Property	CR $\phi_f = 0.62$	WR $\phi_f = 0.45$	CFM $\phi_f = 0.28$
E_{11}, GPa	45.78	34.05	22.32
E_{22}, GPa	7.39	5.27	4.10
G_{12}, GPa	2.79	1.99	1.54
ν_{12}	0.26	0.28	0.30

The next step is to consider the alignment of the reinforcement in the laminae of Table 6. In the case of the Continuous Roving (CR), the reinforcement is aligned in the 1 (x) direction and the values in Table 6 may be used directly to populate the stiffness matrix [Q] using Equations 8 and 9.

$$[Q] = [S]^{-1} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \quad (3)$$

where

$$\begin{aligned} Q_{11} &= \frac{E_{11}}{(1-\nu_{12}\nu_{21})} \quad Q_{22} = \frac{E_{22}}{(1-\nu_{12}\nu_{21})} \\ Q_{12} &= \frac{\nu_{21}E_{11}}{(1-\nu_{12}\nu_{21})} = \frac{\nu_{12}E_{22}}{(1-\nu_{12}\nu_{21})} = Q_{21} \\ Q_{66} &= G_{12} \end{aligned} \quad (4)$$

The elastic constants of the Continuous Filament Mat lamina given in Table 6 are used to generate the elastic constants of a quasi-isotropic lamina that is made up of 4 layers of equal thickness aligned at $0^o/90^o/45^o/-45^o$. Similarly, the elastic constants of the Woven Roving lamina given in Table 6 are used to generate the properties of an orthotropic laminate from 2 layers of equal thickness aligned at 0^o and 90^o.

These parameters may be obtained from the reduced stiffness matrix [A] according to

$$\begin{aligned} E_{xx} &= \frac{A_{11}A_{22} - A_{12}A_{21}}{dA_{22}} \qquad G_{xy} = \frac{A_{66}}{d} \\ E_{yy} &= \frac{A_{11}A_{22} - A_{12}A_{21}}{dA_{11}} \qquad \nu_{xy} = \frac{A_{12}}{A_{22}} \quad \nu_{yx} = \frac{A_{12}}{A_{11}} \end{aligned} \quad (5)$$

The terms in [A] may be calculated by summing the thickness-weighted terms in the transformed stiffness matrix $[\overline{Q}]$ for each constituent layer according to

$$A_{i,j} = \sum_{k=1}^{N} \overline{Q}_{i,j} \cdot t_k \quad (6)$$

The laminate thickness, d, is sum of the thicknesses of each of the N layers that it contains

$$d = \sum_{k=1}^{N} t_k \ . \tag{7}$$

The transformed stiffness matrix for each lamina is given by

$$[\overline{Q}] = [T]^{-1} \cdot [Q] \cdot [R] \cdot [T] \cdot [R]^{-1} \tag{8}$$

In Equation 8, [T] is the transformation matrix (Equation 9) and [R] the Reuter matrix (Equation 10). θ is the angle between the fibre in the particular lamina and the reference (loading) direction of the laminate.

$$[T] = \begin{pmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{pmatrix} \tag{9}$$

$$m = \cos\theta \quad n = \sin\theta$$

$$[R] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \tag{10}$$

The elastic constants for the CFM and WR layers derived from the data in Table 6 and Equations 4-10 are shown in Table 7 [2].

Table 7: Elastic Constants for WR and CFM Laminae

Lamina Property	WR	CFM
E_{11}, GPa	19.79	10.045
E_{22}, GPa	19.79	10.045
G_{12}, GPa	1.99	3.817
ν_{12}	0.0751	0.3159
ν_{21}	0.0751	0.3159

Finally, the elastic constants of the pultruded profile may be calculated using Equations 4-10 from the data in Table 7 and Table 6 (for CR) for a range of thicknesses of each style of lamina. The results of such a calculation for a profile composed only of CFM and CR are given in Table 8.

Table 8: Elastic Constants of a CFM / CR Pultruded Profile

	Thickness Ratio, CFM : Roving								
Constant	**5:1**	**4:1**	**3:1**	**2:1**	**1:1**	**1:2**	**1:3**	**1:4**	**1:5**
E_{11}, GPa	16.0	17.19	18.98	21.96	27.92	33.87	36.85	38.64	39.83
E_{22}, GPa	9.91	9.85	9.75	9.53	9.05	8.51	8.24	8.08	7.96
G_{12}, GPa	3.65	3.61	3.56	3.47	3.30	3.13	3.05	2.99	2.96
ν_{12}	0.313	0.312	0.311	0.309	0.304	0.299	0.297	0.295	0.294
ν_{21}	0.193	0.179	0.159	0.134	0.099	0.075	0.066	0.062	0.059

At this stage it is useful to express the thickness ratio given in Table 8 in terms of the weight fraction or volume fraction of the different laminae in the profile.

This may be accomplished by considering the notion of unit thickness of the reinforcement. This is defined as the thickness of a layer of “solid” reinforcement that weighs 1kg per square metre and is equivalent to the reciprocal of its density (2560kg/m^3). Thus the thickness of a layer that contains reinforcement weighing W g/m^2 at a volume fraction of ϕ_f is, in mm

$$t = \frac{W}{\rho_f \phi_f} \tag{11}$$

It follows from Equation 11 that the thickness ratio for CFM:CR for a constant W, is the inverse of the ratio of the maximum volume fraction of reinforcement in each layer, namely 0.62:0.28 or 2.2:1. The corresponding elastic constants of this lay-up may be calculated using laminate analysis as before and are shown in Table 9.

Table 9: Elastic Constants for a Profile having Equal Weights of CFM and CR based on a Thickness ratio of 2.2:1

Parameter	Value
E_{11}, GPa	21.21
E_{22}, GPa	9.58
G_{12}, GPa	3.49
ν_{12}	0.309

The unit thickness of Equation 11 underestimates the layer thickness found in practice. From experimental measurements made at 80 psi (compare with Table 1) the thickness of various CFM layers are given in Table 10

Table 10: Layer Thickness for CFM Measured at 80psi

W, g/m^2	t, mm
300	0.5
450	0.74
600	0.99

If the cross-sectional area of a single end of fibre having tex T g/km is A (mm^2) then the "thickness" of a layer of continuous roving, t_{CR}, that has a surface weight, W (g/m^2) is given by

$$t_{CR} = \frac{WA}{T} \tag{11}$$

In practice the value of A has been estimated at $2.86mm^2$. Thus the thickness of a CR layer having a surface weight of 450 g/m^2 is 0.27mm. Hence from Table 10, the thickness ratio CFM:CR for an equal surface weight of $450g/m^2$ is 2.74. This exceeds that derived from the unit thickness approach (2.2) by about 25%. The elastic constant of a profile containing CFM and CR at this ratio are shown in Table 11.

Table 11: Elastic Constants for a Profile having Equal Weights of CFM and CR based on a Thickness ratio of 2.7:1

Parameter	Value
E_{11}, GPa	19.6
E_{22}, GPa	9.69
G_{12}, GPa	3.54
ν_{12}	0.31

Using the data from Table 10 and the cross-sectional area of a single end of CR, the precise composition of a profile and its elastic constants may be calculated. The results of such an analysis for a 3.5mm x 100mm plate made up of varying number of layers of $450g/m^2$ CFM and 4800 tex CR are shown in Table 12.

Table 12: Compositions and Axial Modulus of a Pultruded Plate based on CFM and CR.

CFM Layers	CFM Thickness mm	CR Thickness mm	Ends of 4800 tex	Thickness Ratio CFM:CR	E_{11}
1	0.74	2.76	97	0.27	38.18
2	1.48	2.02	71	0.78	30.67
3	2.22	1.28	45	1.73	23.11
4	2.96	0.54	19	5.48	17.03

References

1 J A Quinn, Fibreforce Design Manual, 1988
2 J M Methven, YALAP Laminate Design Program, 1997-1999